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Miles Townes mdtownes@gwmail.gwu.edu

Hierarchy and Complex Systems

In his paper on the emergence of cooperation, Axelrod points to the U.S. Congress as an example of the kind of social situation in which cooperation emerges: "...the diminished turnover rate... can allow the development of reciprocity".[1] While this may have been true of the 1970s and '80s, the last fifteen years or so of Congressional politics have arguably seen a decrease in norms of reciprocity and cooperation despite no increase in the turnover rate. Members of Congress are increasingly polarized by party identity, and at least some of this change is due to the leadership of both parties.[2] Indeed, legislators are not independent agents but are located within hierarchical structures of party, seniority, and caucus affiliation; they must comply with the demands of agents in superior positions in the hierarchy or risk punishment.

Hierarchy is not unique to Congress, and is indeed a common feature of human and animal social systems; a wolf pack has its alpha male, a beehive has its queen, and a Congress has its Speaker, Leaders, Whips, and Chairmen. Yet when these and similar systems are the subject of computational modeling, modelers tend to omit or elide the hierarchical aspect of agent interactions. For most cases, the omission of hierarchy has no consequence; hierarchy does not affect the particular phenomenon of interest. For some cases, however, hierarchy is an essential and irreducible element, especially when the goal of the model is useful inferences about social behavior in systems that are characterized by hierarchy. For many cases, especially concerning political systems, hierarchy is relevant to the broad interpretation of modeling results. Thus it is important that computational modelers be able to identify, describe, and demonstrate hierarchy.

In this paper, I discuss the potential for computational models of hierarchy. First, I begin by

1 Axelrod, Robert. "The Emergence of Cooperation among Egoists". *American Political Science Review*, 75 (1981); p. 309

2 At least, there is a popular perception that Congress is increasingly polarized: for example, Billings, Erin P. "Democrats Adjust Strategy..." *Roll Call* , Jan. 17, 2008; Brownstein, Ronald. "Partisanship today, partisanship tomorrow." *Times Union (Albany, NY),* Oct. 20, 2004; p. A11. (editorial) "The grip of gridlock Congress lacks a sense of proportion, thanks to domination by partisan diehards." *Omaha (NE) World-Herald*, September 6, 2003; p. 6B.

Miles Townes mdtownes@gwmail.gwu.edu

summarizing a relevant and cogent critique of complexity theory, to which this paper is largely a response. Second, I review the literature from natural and social science on hierarchical systems. Third, I describe the components of hierarchy and suggest a basic typology of hierarchical structure. Fourth, I posit mechanisms for the emergence of hierarchy. Fifth, I discuss extant models of social interactions in which hierarchy is potentially a factor but remains latent in the simulation. Sixth, I suggest models that explicitly model hierarchy as an emergent and complex phenomenon. Finally, I discuss the consequences of hierarchy for computational social science, and the prospects for meaningful models of hierarchic social systems.

Before turning to the argument of my paper, I want to clarify what is meant by hierarchy. Unfortunately, the social sciences literature offers very little by way of definitions; most authors take hierarchy as a given or obvious phenomenon. There is almost no literature on hierarchy as an abstraction, thus leaving those who wish to study the abstraction to their own devices.

Thus "hierarchy" in its present usage means the structure of unequal authority within a social group. Hierarchy can also mean the structure of inequality within a group,[3] but the concern here is how inequality empowers some agents at the expense of others within a group.

I. The Earnest and Rosenau Critique

In their chapter of the book, *Complexity in World Politics*, David Earnest and James Rosenau question whether the ontological commitment of complexity theory render it unsuitable for analysis of political situations. They point out that political systems are characterized by authority: actors in these systems exercise influence over one another in ways that restrict or limit their decision-making ability. However:

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3 See, for example, Flanagan, James G. "Hierarchy in Simple 'Egalitarian' Societies". *Annual Review of Anthropology*, 18 (1989); p. 248.

Miles Townes mdtownes@gwmail.gwu.edu

> ... the pattern of authority in a complex adaptive system is one of its distinctive features; it has none. Authority is perfectly decentralized; each agent decides and acts on the basis of internal rules that evolve in response to environmental feedback. This is the logical antithesis of social authority... [which] raises the question of the appropriateness of the complexity metaphor for the study of politics: are authoritative systems logically incompatible with complex adaptation?[4]

Note that they view authority as "perfectly decentralized" in complex systems; it is not that agents possess no ability to influence one another, but that all agents possess the same influence. Authority in the sense that Earnest and Rosenau intend is a term of art in political science; other disciplines, especially natural sciences, use hierarchy to describe roughly the same phenomenon. Thus the focal concept in this paper is not with authority *per se*, but rather with inequality of authority: that is, hierarchy as defined above.

Earnest and Rosenau do not believe that hierarchy ("authority") is entirely beyond the grasp of complexity theory, but they argue that the most relevant way to approach it is to "grow authority from the bottom up, as an emergent property of a complex system".[5] They discuss several attempts to do so, but reject them as unsatisfactory.

> While these approaches are an important first step, they may not truly simulate the emergence of the authoritative allocation of values for a population of agents. ... [Axelrod's] tribute model produces only quantitatively different actors: some states in the model develop more power than others. But the emergent actors are not qualitatively different: they do not make decisions for other agents in the system. Epstein and Axtell's sugarscape model is similarly devoid of agents that are qualitatively different.... Each follows local decision rules.[6]

This is an important but not devastating critique. There are several aspects that merit further discussion, some of which are beyond the scope of this paper. First among these is the ontological claims of complexity theory; for the present, these claims are assumed to be not as stringent as Earnest and Rosenau would admit, or at least not inherent in all forms of computational modeling. We can proceed with either a relaxed version of complexity theory, or with computational modeling only

4 Earnest, David and James N. Rosenau. "Signifying Nothing? What Complex Systems Theory Can and Cannot Tell Us About Global Politics", in Harrison, Neil (ed.). *Complexity in World Politics*. (SUNY: Albany, NY, 2006). p. 153.

5 Ibid, p. 155.

6 Ibid, p. 155.

Miles Townes mdtownes@gwmail.gwu.edu

loosely bound to complexity theory.

A more important set of issues centers on agency and authority in political interactions. International relations as a discipline has recently rediscovered the human as agent in politics, where it was primarily concerned with structures in the past. The challenge to the discipline is whether it can accommodate humans as the ultimate actors---that is, the agents that constitute structure.[7] This is still a controversial perspective in international relations, but it is the only account that is sympathetic to the ontological assumptions of complexity theory. If we begin with the assumption that human agents by their interactions construct macro institutions, we can take authority as one of those institutions and model it as the consequence of agents.

If authority depends on agency, then Earnest and Rosenau are perhaps wrong to imply that authoritative agents "make decisions for other agents"; at least this this is not the only way to describe authority. Authority may be an attribute of specific humans, but it requires a willingness on the part of other humans to defer to or comply with authority. Compliance moderates the extent to which authority is absolute. It is not always true that authority commands automatic compliance; in nearly every political interaction, the non-authoritative agent possesses the option of non-compliance. In many situations, non-compliance can result in severe, possibly terminal consequences, but coercion is not always absolute. To capture this distinction, it is perhaps more helpful to say that authoritative agents affect the decisions of other agents, in that subordinate agents must include superior agents' decisions in their own decision-making. Here the definition of hierarchy as inequality of authority proves useful by permitting degrees of authority in social system, as opposed to a simple binary characteristic of agents. Authority might be simply the result of a quantitative difference in compliance to various agents; whether or not this view rehabilitates the models Earnest and Rosenau criticize depends on the extent to

7 See, for example, Wight, Colin. “State Agency: social action without human activity”. *Review of International Studies*, 30 (2004).

Miles Townes mdtownes@gwmail.gwu.edu

which those models explicitly invoke authority and compliance.

Despite their critique, Earnest and Rosenau are sympathetic to the possibility of computational models of hierarchy. The goal, they argue, should be to produce artificial authorities that behave much as human authorities. To that end, they propose a test of artificial authority as validation for future models:

> If and when a complexity researcher succeeds in generating authority from the bottom up in a complex adaptive social system, the researcher should rewrite the program so that human subjects can participate as if they were agents within the system.... when human subjects comply with the system's authority in the same manner as the silicon agents do, then one may reasonably argue the system is authoritative.[8](158)

This is a high bar to pass, but not an impossible one. It does require a more concrete understanding of what is meant by hierarchy and how it affects social interactions.

II. Literature on Hierarchy

Natural and social science provide many examples of hierarchical relationships. Among animals, hierarchic ordering has been observed in passerines (birds),[9] wolves,[10] and ants.[11] For example, most passerines (a kind of bird) maintain linear hierarchies, but some flocks manifest non-linear triads:

8 Earnest, David and James N. Rosenau. "Signifying Nothing? What Complex Systems Theory Can and Cannot Tell Us About Global Politics", in Harrison, Neil (ed.). *Complexity in World Politics*. (SUNY: Albany, NY, 2006). p. 158.
9 Lahti, Kimmo; Kari Koivula; and Markku Orell. “Is the Social Hierarchy Always Linear in Tits?” *Journal of Avian Biology*, 25:4 (Nov. 1994); pp. 347-348.
10 Fox, M. W. “The Social Significance of Genital Licking in the Wolf, Canis Lupus”. *Journal of Mammalogy*¸53:3 (Aug 1972), pp. 637-640.
11 Cole, Blain J. “Dominance Hierarchies in Leptothorax Ants”. *Science*, New Series, 212:4490 (April 3, 1981), pp. 83-84 .

> In Flock A, the two juvenile males (jm_1 and jm_2) and one of the two adult females (AF_1) formed a triad with a nonlinear social hierarchy. Male jm_1 dominated jm_2, but was subordinate to the adult female (AF_1) who in turn was dominated by jm_2…. Therefore these three birds (jm_1, jm_2, AF_1) were each subordinate to two, and dominated three, individuals in the flock.[12]

Presumably hierarchy offers some evolutionary advantage in passerine flocks, but it is not clear what purpose it serves nor from where it might originate. Passerines are not the only bird that demonstrates hierarchy; one author calls chickens "the premier animal of dominance studies".[13]

In a novel study of green swordtails (fish), researchers tested whether "winner effects" influenced hierarchy formation. A "winner effect" occurs when the winner of an aggressive interaction gains an internal advantage in future interactions; likewise, a "loser effect" creates an internal disadvantage in future interactions. To test this, the researchers put swordtail specimens of approximately the same size into pairwise competition to create winners, losers, and neutrals. These fish were then separated and combined with fish from other competitions; if "winner effects" were irrelevant, then the size-matched fish could be expected to win or lose equally in subsequent interactions. However, this did not happen; winners were more likely to emerge as winners and losers more likely to remain losers in subsequent interactions. The authors conclude,

> These results strongly suggest that information on intrinsic measures such as size and resource holding power are not sufficient to predict hierarchy formation... Instead, a combination of intrinsic factors, with the extrinsic factors measured here (i.e. winner and loser effects) are necessary to make detailed predictions regarding hierarchy formation.[14]

This study has important consequences for how we think about hierarchy and where we locate authority in our models of social systems.

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12 Lahti, Kimmo; Kari Koivula; and Markku Orell. “Is the Social Hierarchy Always Linear in Tits?” *Journal of Avian Biology*, 25:4 (Nov. 1994); pp. 347.
13 Chase, Ivan D. "Social Process and Hiearchy Formation in Small Groups: A Comparative Perspective". *American Sociological Review,* 45:6 (Dec. 1980), p. 909.
14 Dugatkin, Lee Alan and Matthew Druen. "The Social Implications of Winner and Loser Effects". *Proceedings: Biological Sciences,* 271: Supp. 6 (Dec. 7, 2004), p. S489.

Miles Townes mdtownes@gwmail.gwu.edu

Another useful study focused on ants, a species often modeled as decentralized. Ants are widely known to possess differentiation among colony members: a queen who is responsible for reproduction, workers who gather food and build the nest, and in many species soldiers who guard the colony. This is functional specialization, not hierarchy. However, unique among ant species is *Lepthothorax allardycei,* colonies of which demonstrate hierarchic interaction among workers. The workers, like the queen, are all female, and some contribute to reproduction of the colony. Which workers make that contribution is the result of hierarchical interactions: "Reproductive competition among the workers is resolved through the formation of a dominance hierarchy involving routine displays of dominance, avoidance behavior, and even fighting".[15] The advantage to the ant colony as a society from this interaction is unknown, but ants at the top of the hierarchy enjoy two advantages; first, "high-ranking workers receive food from low-ranking workers but do not reciprocate"; second,

> It can be argued that there are two reproductive castes: the queen, who is responsible for production of all the females, and the high-ranking workers, who account for at least a significant proportion of the male population.[16]

Reproductive fitness also underlies a view of hierarchy among mammals.

> In populations structure by behavior, animals are exposed to a hierarchical arrangement of individuals that share varying proportions of their genome.... Behaviors directed toward individuals in different hierarchical levels will vary in their genetic rewards and their associated risks and benefits. Evolution of altruistic behavior and cooperation among kin has been based on an understanding that behaviors favor individuals that forfeit a part of their fitness to benefit others that share a fraction of their genome IBD [identical by descent].[17]

As one moves along the evolutionary tree towards primates and ultimately humans, hierarchy becomes more common and also more complex. While primate groups often exhibit linear hierarchy, they also form complicated structures like coalitions and dependent rank: "In dependent rank, a particular individual (animal) is able to achieve dominance over another animal in the presence of a third animal

15 Cole, Blain J. "Dominance Hierarchies in Leptothorax Ants". *Science*, New Series, 212:4490 (April 3, 1981), pp. 83.
16 Cole, Blain J. "Dominance Hierarchies in Leptothorax Ants". *Science*, New Series, 212:4490 (April 3, 1981), pp. 83-84 .
17 Chesser, Ronald K. "Relativity of Behavioral Interactions in Socially Structured Populations". *Journal of Mammalogy*, 79:3 (Aug. 1998), pp 713-724.

Miles Townes mdtownes@gwmail.gwu.edu

(frequently its mother or consort) but not able to do so if the third animal is not present".[18]

Unfortunately, the natural science literature offers few starting points for an understanding of hierarchy and little in the way of direction. Fortunately, Ivan Chase (a sociologist) has produced a thorough survey of the literature on animal and human studies of hierarchy. He suggests that most explanations of hierarchy fall into one of two categories, "a correlational model and a pairwise interaction model".[19]

> Explanations that are basically correlational models are those that indicate that individuals' positions in a hierarchy are determined by their physical attributes, their genetic endowment, their hormonal state, their past social performance, their personality traits, the social labels that they have been given, or any composite of these factors....
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> Explanations of hierarchies that are pairwise interaction models in their canonical form are, for example, differences in fighting ability for animals, theories dealing with the interaction of personality types for humans, and exchange theories for humans.[20]

Chase finds that the mathematical conditions necessary to generate both models in theory are far too stringent compared to reality, thus previous treatments of hierarchy formation are inadequate. He suggests analysis based on dominance triads, on which more below.

The above studies suggest several important inferences for computational models of hierarchy. First, hierarchy does in fact emerge from complex systems of simple (i.e. non-human) agents. This permits computational scientists to test simple models against empirical evidence from simple agents, before embracing human hierarchy in all its complexity. This leads to the second inference, that there is in fact significant room for computational models in accounts of animal hierarchy. As mentioned, zoological reports of hierarchy often cannot explain the origins or advantages of hierarchy except in the

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18 Chase, Ivan D. "Social Process and Hiearchy Formation in Small Groups: A Comparative Perspective". *American Sociological Review,* 45:6 (Dec. 1980), p. 906.
19 Chase, Ivan D. "Social Process and Hiearchy Formation in Small Groups: A Comparative Perspective". *American Sociological Review,* 45:6 (Dec. 1980), p. 908.
20 Chase, Ivan D. "Social Process and Hiearchy Formation in Small Groups: A Comparative Perspective". *American Sociological Review,* 45:6 (Dec. 1980), p. 908-909.

Miles Townes mdtownes@gwmail.gwu.edu

broadest terms. Computational science offers traction on these questions through simulation of the movement of a society of agents from simple, de-centralized order to a hierarchical structure.

The third inference is that hierarchy is a pervasive feature of social, especially human systems. It is therefore essential to the progress and productivity of computational social models that they address, at least tangentially, hierarchy. There may well be human interactions which can safely be assumed to be non-hierarchical and egalitarian; as a review of the anthropology literature suggests, "even rigidly hierarchical systems may contain egalitarian subsystems, just as so-called egalitarian systems may contain insidious hierarchies".[21] The fact that societies contain subsystems is crucial; it means, for example, that models of bird flocking behavior need not account for the presence of dominance hierarchies to be compelling. Likewise, extant models of ant foraging can be useful in understanding aspects of *L. Allardycei* behavior without taking into account that species' reproductive structure. However, if computational science focuses only on those situations it will never offer anything more than partial and biased perspective on human interaction.

III. Social Models with Insidious Hierarchy

In fact, there are already a number of models which focus on social situations that have "insidious" hierarchy. Where such models are "caricature" of reality - Level I in Axtell and Epstein's schematic[22] - this is not problematic. But as models increase in sophistication and especially attempt "quantitative agreement with empirical macro-structures", the problem of latent hierarchy becomes

21 Flanagan, James G. "Hierarchy in Simple 'Egalitarian' Societies". *Annual Review of Anthropology*, Vol. 18 (1989), p. 262.

22 Axtell, Robert and Joshua Epstein. "Agent-Based Modeling: Understanding Our Creations". *Bulletin of the Santa Fe Institute*, Winter 1998. p. 28.

Miles Townes mdtownes@gwmail.gwu.edu

more difficult. This is especially true of models that are intended to aid and inform real-world decisions and actions.

For example, in their book on complex systems, Miller and Page discuss the "Standing Ovation" problem at length and extend a simple version of the model to include seat placement and acquaintances. Most theaters are designed so that the audience faces generally the same direction; thus, conclude Miller and Page, "those in the front have the most visual influence on others, yet also have the least visual information; whereas those in the back with the most information have the least influence".[23] This is a caricature of a minor problem, but it nonetheless omits the fact that seats closer to the stage are often the most expensive and prestigious. Thus seat placement functions as a form of hierarchy in this problem. The consequence is that the cost for the people in the very front to stand up and applaud is low, as is the cost to remain seated if everyone behind them is seated. But moving away from the stage, the cost for being the first or last to stand up is somewhat higher; in the former case, one risks looking like an overly enthusiastic dilettante; in the latter, an uncultured philistine.[24] Indeed, casual observation suggests that most standing ovations initiate in the front of the theater and propagate to the rear. Miller and Page admit that standing ovations are a trivial problem, but also suggest that they point to a broader set of problems in which "people get tied to, and are influenced by other people".[25]

A more consequential example comes from Schelling's segregation model, in which agents' preferences for like neighbors drive a heterogeneous community to separate into homogeneous clusters.[26] While this may be a compelling explanation for some kinds of social processes, there is no doubt that real-world segregation in America was largely the result of social norms or formal policies about the interactions of whites and blacks. Research from as recently as 1989 shows that real estate

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23 Miller, John and Scott Page. *Complex Adaptive Systems*. (Princeton: NJ, 2007), p. 13.
24 Another factor, which also works in favor of hierarchy, is that some people are simply lazy and refuse to stand unless they are otherwise forced to stare at the rear ends of the people in front of them.
25 Miller, John and Scott Page. *Complex Adaptive Systems*. (Princeton: NJ, 2007), p. 14.
26 Schelling, Thomas. *Micromotives and Macrobehavior*. (Norton; NY, 2006 (1978)); see esp. Ch. 4.

Miles Townes mdtownes@gwmail.gwu.edu

agents help perpetuate segregation through their interactions with minority clients, for example by steering them towards towards properties in neighborhoods of the same racial make-up.[27] In these interactions, the real estate agent possesses authority by virtue of their access to information about available properties; it is this access that the client pays for when using an agent to find a house. This access also allows to real estate agent to discriminate against their client by provided only selected information about available properties.

Schelling does assert that "organized action" is beyond the scope of his inquiry,[28] putting real estate agents and zoning regulations beyond the purview of his model. If the model is simply intended as an abstract demonstration with a narrative attached, this is not problematic. If, however, the model is intended to have any bearing on the problem of segregation in the real world, there are two problems. First, it may well be the case that people want to live in segregated neighborhoods, and that only a very small portion of people have the sorts of preferences that Schelling postulates. In that case, the interesting question is not how individual preferences aggregate into suboptimal outcomes, but rather how preferences are formed and reinforced such that apparently suboptimal outcomes are in fact optimal to the agents themselves. Second, if "organized action" does enforce segregation, then Schelling's model is trivial; no matter what the preferences of the actors, the institutional structure of society will perpetuate segregation indefinitely. Policies designed to work from the bottom-up - to encourage people to live in diverse neighborhoods - are likely to fail if real estate agents are allowed to direct clients into same-race neighborhoods.

IV. Conditions and Types of Hierarchy

27 Ondrich, Jan; Alex Strickler; and John Yinger. "Do Real Estate Brokers Choose to Discriminate? Evidence from the 1989 Discrimination Study". *Southern Economic Journal*, 64:4 (April 1998), pp. 880-901.

28 Schelling, Thomas. *Micromotives and Macrobehavior*. (Norton; NY, 2006 (1978)); p. 138.

Miles Townes mdtownes@gwmail.gwu.edu

If models of social and political systems are to address hierarchy, it is helpful to know what circumstances allow or encourage the emergence of hierarchy. As defined, hierarchy is the inequality of authority among a group of agents. An important distinction is that between hierarchy as endogenous structure and hierarchy as exogenous ordering of agents. Many systems and models generate outcomes that can be classified *ex post* as hierarchies - for example, ranking agents by the wealth accumulated in an economic game. To the extent that this sort hierarchy is an external ranking, with no importance to the agents actually playing the game, it is exogenous. If the ranking is relevant to play of the game, then the hierarchy is endogenous. The present work is concerned exclusively with endogenous hierarchy.

Although hierarchy is a structure of systems, the ability to generate hierarchical structures resides within agents; hierarchy emerges from their interactions. This ability in turn depends on two factors. First is the differentiation of agents from one another: "Inequalities develop through evolutionary differentiation".[29] If agents have no diversity in any of their attributes, then there is no criteria by which they can be ranked. The attributes by which agents differ may be instrumental, providing them with greater fitness against other members of the society or within the environment. It might be familial, in that agents possess distinct kin lineage that inform the structure of hierarchy. The attribute may also be entirely arbitrary, such as markings or coloration, which provide no clear survival advantage but may evolve within certain groups or species as indicators of hierarchical position.

The second factor necessary to hierarchy is the social awareness of each agent. Agents must be able to recognize not only their own place in the hierarchy, but also the position others hold. A simple transitive relationship in which each agent knows that it is subordinate to one agent and superior to

29 Flanagan, James G. "Hierarchy in Simple 'Egalitarian' Societies". *Annual Review of Anthropology*, Vol. 18 (1989).

another is insufficient; the agent must be able to place other agents in the social hierarchy. In a sense, agents must maintain their own model of the social hierarchy, and be able to place in that model all agents with whom they interact.

In linear hierarchy, agents hold rank in a continuous order with no treeing or branching (see Figure 1c); among birds, linear dominance hierarchy means "an individual dominates all the birds below it and is subordinate to all the birds above it in the hierarchy. Only one individual occupies each step of the ladder**".** [30] This is the simplest form of hierarchy, and as discussed above, it is frequently observed among animals. For example, some species of monkeys recognize a linear dominance hierarchy, "[which] accurately predicts the direction of competitive interactions between any two individuals".[31]

Figure 1: Three Hierarchical Structures

a. Linear Hierarchy	b. Treed Hierarchy	c. Complex Hierarchy

In treed hierarchy, agents form a tiered or branched network, with one agent at apex and linear order through branches (see Figure 1b). Agents dominate all the agents beneath them in their branch, but not all the agents in lower positions in other branches. This is perhaps the most familiar sort of hierarchy, especially in human social systems; for example, it is the sort of hierarchy described by the phrase

30 Lahti, Kimmo; Kari Koivula; and Markku Orell. "Is the Social Hierarchy Always Linear in Tits?" *Journal of Avian Biology*, 25:4 (Nov. 1994); p. 347

31 Seyfarth, Robert M. and Dorothy L. Cheney. "Social Awareness in Monkeys". *American Zoologist*, 40:6 (Dec. 2000), p. 904.

"chain of command" in military organizations. Treed hierarchies can be modeled as a series of lines, each with the same initial agent and different terminal agents.

Linear hierarchy can be thought of as a special case of treed hierarchy, in which the tree has only one branch (i.e. line). This accords with Chase's suggestion that hierarchy be analyzed in terms of triads of agents:

> ...in a linear hierarchy all possible triads have transitive dominance relationships, and if a hierarchy is not linear it contains at least one triad with intransitive dominance relationships-and the fewer the intransitive triads, the closer the hierarchy to linearity.[32]

By this definition, what I call linear and simple treed hierarchies can both be considered linear.

In complex - or non-linear - hierarchy, agents form a tree-like network, but with variations that make it difficult to trace linear patterns of dominance throughout the branches (see Figure 1c); recall that this was observed in bird flocks by Lahti (fn. 8 above). This kind of triad is described by the vertices H, B, and D in Figure 1c. Other potential complications in complex hierarchy include multiple agents at the top-most level (H, A, and I), multiple paths to the same agent (as shown by the paths A→B→F and A→C→F in 1c), and level-jumping dominance (I→G).

Chase also offers a useful tool for analyzing hierarchies: maps of interactions can be converted into matrices of dominance. For example, the linear hierarchy in Figure 1b can be described in the matrix in Figure 2. A one indicates that the row agent dominates the column agent; a zero indicates no dominance. Thus agent A dominates three others - B, C, D - and agent D dominates none. The matrix tells us that the hierarchy is linear because there are no ones beneath the diagonal. Similarly, Figure 3 shows the matrix for Figure 1c; it is non-linear because there are ones beneath the diagonal. The use of matrices to depict hierarchy offers an advantage to computational modelers, both in defining the

32 Chase, Ivan D. "Social Process and Hiearchy Formation in Small Groups: A Comparative Perspective". *American Sociological Review,* 45:6 (Dec. 1980); p. 909.

Miles Townes mdtownes@gwmail.gwu.edu

exogenous structure of hierarchy for certain models and recording and reporting the emergence of hierarchy in other models.

Figure 2. Pair-wise Dominance Matrix for Linear Hierarchy

	A	B	C	D
A	-	1	1	1
B	0	-	1	1
C	0	0	-	1
D	0	0	0	-

Figure 3. Pair-wise Dominance Matrix for Complex Hiearchy

	A	B	C	D	E	F	G	H	I
A	-	1	1	1	1	1	1	0	0
B	0	-	0	1	1	1	0	0	0
C	0	0	-	0	0	1	1	0	0
D	0	0	0	-	0	0	0	0	0
E	0	0	0	0	-	0	0	0	0
F	0	0	0	0	0	-	0	0	0
G	0	0	0	0	0	0	-	0	0
H	0	1	0	1	1	1	0	-	0
I	0	1	0	1	1	1	1	0	-

One further distinction is necessary to a complete this discussion: pure hierarchy versus noisy hierarchy. If in a hierarchy, information is transmitted among the agents with no loss as it moves from top to bottom, this may be called a pure hierarchy. Where transmission is subject to noise or random error, this may be called a noisy hierarchy. As an example of noisy hierarchies, consider the parlor game Telephone, in which a short message is whispered from player to player until it has passed through every person in the group; often the message is distorted to a degree related to the number of players. In fact, some degree of noise is typical of hierarchies in natural and social systems; human and animal interactions are often subject to miscommunication, misinterpretation, bias, random error, and other sorts of noise. Noisy hierarchy can also be expressed in matrix form, using fractional coefficients to indicate each step (see Figure 4).

Figure 4. Linear Hierarchy with .05 Noise Coefficient

	A	B	C	D
A	-	.95	.9025	.8574
B	0	-	.95	.9025
C	0	0	-	.95
D	0	0	0	-

V. Hierarchy in Computational Models

The natural and social science literature clearly points to hierarchy as an outcome of complex social interactions, yet offers little explanation of the origins of hierarchy. Most such research takes hierarchy as given and exogenous to the society, without explain its sources or benefits. In consequence, what follows is necessarily speculative and abstract.

Computational models wishing to account for hierarchy as a given condition of social interaction can do so easily by including a matrix of pair-wise interactions and coefficients to indicate the structure of the system. This is akin to techniques used in network analysis and thus familiar to computational social scientists. The effects of hierarchic structure on social models is an area ripe for research.

The more challenging problem for modelers is to account for the emergence of hierarchy from social interactions. The lack of a clear mechanism for hierarchy in the literature suggests it is likely that

Miles Townes mdtownes@gwmail.gwu.edu

there is no single mechanism, but that it is the consequence of any number of evolutionary processes. Despite Chase's misgivings of correlation and pairwise interaction as explanations for hierarchy, these concepts may yet be useful for thinking about evolutionary processes that generate hierarchy.

I posit two possible processes, which are by no means exhaustive of all possibilities. First, hierarchy can arise through imitation - somewhat akin to correlation. Consider a coordination game in which agents possess a range of strategies as they interact. If agent A is more successful than agent B, then B may choose to switch its strategy to that of A. Assuming that the game is not zero-sum (that is, B's adoption of A's strategy does not diminish A's payoff), this in effect creates a hierarchical relationship between the two agents. The relationship is entirely voluntary, as neither agent can compel the other to sustain it, and thus the hierarchy is likely to be fragile. Hierarchy by imitation is the weakest notion of hierarchy considered here.

Second, a coercive hierarchy emerges when one agent does possess the ability to compel the obedience of other agents, akin to pairwise interaction. In effect, the dominant agent is so strong that he can change the payoffs for the other players. Consider again the coordination game, in which one agent has accumulated significant wealth by chance or design. If that player is wealthy enough, it can offer side payments to other players to ensure it receives its desired outcome; in effect, it uses its wealth as power to change the payoffs. If the other players are rational, the change in payoffs compels them to adjust their strategies accordingly.

To extend our understanding of hierarchy, I propose (but do not implement) a model of legislative activity. Within political parties in the U.S. Congress (and other parliamentary bodies) there is often tension between the legislative agenda as a macro-structure and the interests of members' constituents as micro-motives. The agenda is not exogenous; it depends on the rules that grant agenda-making power to members with seniority. Thus the agenda reflects the senior members' interests at the

Miles Townes mdtownes@gwmail.gwu.edu

expense of junior members. Based on this, I suggest a model with the following rules:

L1. A population of agents possesses preferences given exogenously, in the form of random numbers.

L2. Every turn, the simulation generates a random number.

L3. Agents vote for the number if it is sufficiently similar to their preferences, or against if it is dissimilar.

L4. Periodically, agents voting record and (exogenous) preferences are compared. If the record and preferences are not sufficiently aligned, the agent is terminated and a new agent takes its place.

In the game so far, agents have no incentive to stray from the exogenously given preferences (i.e. the will of their constituents) and none will be terminated. I posit that this game results in trivial outcomes unless two further rules are included.

L5. Agents gain seniority as they remain longer in Congress, which translates to higher position in the hierarchy.

L6. Highly ranked agents affect the process by which random numbers are generated.

I anticipate that over time the model will select for a narrow agenda based on a few agents who have maintained seniority. Junior agents will find their existence in the system very precarious as they try to reconcile their preferences with the legislative agenda.

This basic model can be further expanded to include a number of variants. First, the preferences can be modeled as normally distributed, bipolar, or multi-polar. Second, preferences can be modeled as multi-dimensional, with several "issues" on which members vote, instead of the single random number in the basic model. Third, agents can possess a variety of explicit voting rules, or voting rules can be subject to a genetic algorithm. Properly implemented, this model would also have the benefit of lending itself to the sort of human subject validation that Earnest and Rosenau propose. A group of students could play alongside simulated agents to determine whether the model develops genuine authority.

Miles Townes mdtownes@gwmail.gwu.edu

VIII. Prospects for Hierarchy in Computational Social Science

Animal and especially human systems frequently demonstrate hierarchic interactions within groups; this is an empirical fact which computational social science has not yet addressed in a satisfactory way. If the discipline is shed light on social and political relations of humans, we must develop conceptual tools that allow us to describe and analyze hierarchy. In this paper, I have suggested ideas that might help to push us in that direction, however incrementally.

When complexity theory is strictly focused on the properties of decentralized agents, then hierarchic systems are by definition outwith the theory and we face an impassable boundary to our work. Even when this is not the case, the agent focus of modeling techniques predisposes modelers to ignore hierarchic interactions. First, the tools themselves encourage modelers to think of society as fundamentally decentralized. Second, the tools make it difficult to model hierarchy when it is taken into account. Models tend to show how agents create structure, but not how the structure in turn affects the agents' decisions. That does not mean that agent-based modeling is unsuitable to hierarchic systems, but that modelers of such systems should account for the structure of the system explicitly.

While it is preferable that computational science account for hierarchy as a parameter of social phenomena, there is always the option of treating hierarchy simply as a error in our models. This is, in fact, the present default; because when we do not invoke hierarchy as a driving mechanism, whatever effect hierarchy has appears in our models as error in the form of divergence from empirical data. The stronger the hierarchy within the society we are modeling, the greater divergence we are likely to see between our result and the real process. The researcher must then decide whether that divergence is significant or negligible, but this will become a growing problem as models become more sophisticated and empirically grounded.